

Hunting for the Slichter mode in strainmeter records in Japan

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Abstract

The Slichter mode is the gravest eigenoscillation of the Earth, and is a signature of rigid translational motion of the solid inner core relative to the surrounding fluid outer core. We search for the Slichter mode triplets in long period strainmeter records of the 2004 Sumatra-Andaman Islands Earthquake at four Japanese stations. Our approach is to identify time decaying peaks in running power spectra in sub-millihertz band in which the theoretically predicted frequency of this mode resides. After correcting for the Earth tide signals and disturbances by atmospheric pressure fluctuation, we compute running power spectra for moving 6 day long windows for the consecutive 19 days from December 20, 2004 to January 8, 2005. We are able to identify triplet decaying spectral peaks of the Slichter mode that were excited by this earthquake on December 26, 2004 in the 0.05 mHz band in extensometer strain records at one station, Matsushiro Seismological Observatory. Apparent Q of the observed mode is around 15. Power spectral amplitudes of the observed triplet are close to that of the noise level and are consistent with an order of magnitude of the theoretical prediction. Frequencies are consistent with the theoretical prediction for Earth model 1066A (Gilbert and Dziewonski, 1975) than with those for PREM (Dziewonski and Anderson, 1981), implying that the density jump at the inner-outer core boundary is close to the 0.87 g/cm^3 of 1066A rather than the 0.60 g/cm^3 of PREM.

1. Inner Core and the Slichter mode

The inner core represents less than 1% of the volume of the Earth, its mass being approximately 1/60 of the total mass of the Earth. Since its discovery by Inge Lehmann in 1936, seismologists have been trying to understand this small portion of the deep Earth. Existence of the inner core is widely accepted as the consequence of the solidification of the liquid core during the cooling of the Earth. Density of the core is one of the key parameters, as this directly reflects composition of the inner core. Solid inner core is supposed to be less rich in light elements such as hydrogen and sulfur, and a larger density contrast at the inner core boundary (ICB) implies that outer core stores more such light elements. From a thermodynamic calculation, Stacey and Stacey (1999) suggested that a small density contrast at the ICB in PREM (Dziewonski and Anderson, 1981), 0.60 g/cm³, would imply a rapid growth of the inner core at its early stage and the age of the inner core to be 2.3 billion years, which is unfavorable to explain the appearance of the magnetic field at 3 billion years ago or earlier (e.g., Yoshihara and Hamano, 2004).

Our current estimates of the inner core density have come from both high and low frequency seismology. High frequency seismology is good at determining the density jump at the ICB, as major tools are the reflected phases at the ICB. Amplitudes of the reflected phases are controlled by the impedance contrasts at the discontinuity, especially in the case of near-vertical incidence. Since the ICB reflected P phase, PKiKP, has a small amplitude at the near-vertical incidence, its amplitude has been compared to that of the core-mantle boundary (CMB) reflected P phase, PcP. Previous estimates of the density jump at ICB were therefore affected by the assumptions such as the shear wave velocity at the ICB. Reported values in such amplitude ratio analysis ranged from 0.5 to 1.0 g/cm³ (e.g., Cao and Romanowicz, 2004; Koper and Dombrovskaya, 2005).

Low frequency normal modes have energies in the deep Earth and are sensitive to the volumetric average of the shear wave velocity and density of the core. In the 1970's, the inner core density was estimated through determination of eigenfrequencies of low frequency normal modes that have oscillation energy in the inner core, such as ${}_0S_0$ (0.814 mHz, 1228 s), ${}_1S_0$ (1.631 mHz, 613 s), ${}_0S_2$ (0.309 mHz, 3233 s) to ${}_0S_5$ (0.840 mHz, 1190 s). Hereafter we refer to theoretically predicted eigenfrequencies of PREM for the seismic normal modes unless otherwise noted. These modes are sensitive to the velocity and density in the mantle and the outer core as well, and resolving the inner core density from normal mode frequencies requires several assumptions. Estimates of the density jump at the ICB in normal mode study nevertheless agreed well and were between 0.6 and 0.8 g/cm³ (Kennett, 1998; Masters and Gubbins, 2003), which were also comparable with estimates in recent body wave studies.

The other possible tool to determine the density jump at the ICB is the Slichter mode.

Slichter (1961) was the first to predict the rigid translational motion of the inner core relative to the fluid outer core whose major restoring force is the negative gravity buoyancy due to the density jump at ICB. This motion is an eigenoscillation of the Earth, and is denoted as 1S1 in the conventional normal mode notation. Eigenfrequency of this gravity driven Slichter mode is directly controlled by the density jump at the ICB. Owing to Earth's rotation, the degenerate Slichter eigenfrequency is split into three eigenfrequencies, which range from 0.04 mHz (7 hours) to 0.07 mHz (4 hours) (Table 1).

Observing a weak signal from the deep Earth is a challenging task. The difficulty increases with practical problems in time series analysis as our task is to retrieve very low frequency signals from a long record. In Fourier spectral analysis, theoretical spectral resolution improves by square root of the frequency of the signal when the sampling rate is fixed (Zürn, 1974). On one hand, a long time domain window of any geophysical observation is contaminated by both cyclic and non-cyclic noises; examples of the former include the Earth tides whose dominant periods are of hours, and the source of the latter includes activity in the troposphere, i.e., weather. In the long history of extensive search for the Slichter mode, previous claimed observations (Melchior and Ducarme, 1986; Manshinha et al. 1990; Smylie, 1992, Smylie et al., 1993 and Courtier et al., 2000) have been still controversial. Recent studies with the highly sensitive superconducting gravimeters (e.g., Rosat et al., 2006) provided inconclusive results.

Novelty in our study to detect the Slichter mode is to utilize a new type of data, strainmeter records. Several strain stations have been running in Japan to monitor crustal movements under the earthquake prediction program. Though detecting weak signals from the deep Earth is not intended when installed, high sensitivity in this frequency band would make such sensors favorable devices in recording exotic signals such as the Slichter mode. Our sensors are extensometer and Ishii-type borehole strainmeter, both of which are highly sensitive instruments at very low frequencies by design. Principle of the extensometer is as follows; a long bar of quartz crystal is horizontally suspended in a quiet vault, one end of which is fixed to the ground and the other free. The horizontal displacement of the free end to the ground divided by the bar length is the strain value we obtain. The other is Ishii-type strainmeter (Ishii et al., 2002; Asai et al., 2005) which measures deformation of a vessel placed in a borehole that is equipped with magnification devices and magnetic sensors. Their sensitivity is theoretically flat down to 0 Hz (DC component). This is an advantage over a typical broadband seismometer, as sensitivity of the latter falls off below 0.01-0.03 Hz due to the galvanometer mechanism, and seismometers are accordingly less likely to sense any motion in submilihertz band.

In our spectral analysis, we attempt to detect the triplet peaks in the very low frequency band of strain records of the 2004 Sumatra Andaman Islands earthquake which occurred on December 26, 2004. This Mw 9.3 (Stein and Okal, 2005) event was the largest seismic event in

the modern era of geophysical observation, which provided the unprecedented rare opportunity to hunt for the Slichter mode.

2. Data and Spectral Analysis

We use continuous strain recording at four stations, MSO, AMGS, ESH and BYB (Table 2). These stations are chosen among crustal movement stations in Japan for the noise levels in the period band of our interest of hours, being lowest. Sensors at MSO, ESH, and AMGS are extensometers. BYB is a borehole station, and the sensor is an Ishii-type strainmeter. Sampling rates and minimum resolutions of strain amplitudes vary from station to station (Table 2). Figure 1 compares the first 6 hour records of the 2004 Sumatra-Andaman Islands earthquake at the 4 stations. These four traces are records for the highest S/N component at the respective stations. Long period (200-300 seconds) surface waves which traveled around both minor and major arcs were recorded clearly, and subsequent wave packets which traveled around the great circle more than once were also recorded. Amplitudes of the wave packets were similar among four stations.

We first compute running spectra to detect the normal modes. This is a necessary step to learn the signal to noise ratio of our data in the conventional mode frequency band, which is supposedly higher than in the Slichter mode band. As our final goal is to resolve the eigenfrequencies of the Slichter mode triplets, which requires a frequency resolution of 0.002 mHz, we use a time domain window of 2^{19} seconds (524288 s, 6.07 days) in the spectral analysis in this study. From the 19.07 day record from December 20, 2004 to January 8, 2005, we make 312 sets of time series of 6.07 day length with a mutual time lag of 1 hour. After removing the linear trend, we apply the Hanning taper to the respective sets and obtain power spectra from the Fourier spectra estimated with FFT. The 312 sets of power spectra are pasted up along the time axis to obtain a running spectrum. Shown in Figure 2 are running spectra below 2 mHz (500 seconds) at four stations for the respective components shown in Figure 1. A common color scale is used in the four panels. Note that the spectrum in the first column on the left side of each panel is that for 6.07 day long set from 00:00:00.0 UTC, December 20 to 01:38:08.0 UTC, December 26. Prior to the 2004 Sumatra-Andaman Islands earthquake, we had an earthquake of Mw8.1 at Macquarie Islands region on December 23. Disturbance of the spectra (i.e., spectral peaks of excited modes) by this event is unrecognizable in this color scale.

We can identify spectral peaks of many normal modes excited by the 2004 Sumatra-Andaman Islands event. These spectral peaks decay to the noise levels within a few days, which is consistent with that a quality factor Q of these modes are between 300 and 500 as modeled in PREM. Power spectral peaks amplitudes of identified peaks are of the order of 10^{-9} to 10^{-10} strain²s².

Given that the observed power spectral amplitude, SA , of a mode is convertible to its initial amplitude, SI , of an eigenfrequency of f through a relation $SA/SI = Q/(\pi f)$, the initial amplitudes of these modes are estimated as of the order of 10^{-9} to 10^{-10} strain, roughly consistent with the moment of 4×10^{22} Nm of Harvard CMT and 1.2×10^{23} Nm of Stein and Okal (2007).

Noise levels at MSO and ESH are of the same order or are slightly lower than at AMGS

and BYB.

The low noise level is yet low enough that we are able to detect low order modes as seen in Figure 2. Considering the low noise level and high spectral resolution (Table 2), we focus on MSO in the following of this report. Noise levels at the stations are not stationary. Figure 3 shows running spectra of the strains and the atmospheric pressures in the observation vault at MSO. Higher noise level at around 144-th hour is disturbance due to a severe low pressure system that was moving eastward in the western Pacific, south of the Japanese Islands, on December 29. Amplitude of this disturbance is larger in the NS component than the EW component. EW-NS component, the difference between the EW and NS components, represents the horizontal pure shear strain, and should be less sensitive to atmospheric pressures. These appear to be true in our data, as spectral peaks of σT_3 (0.586 mHz, 1706 s), σT_4 (0.766 mHz, 1306 s) and σT_5 (0.929 mHz, 1076 s), which are weak in EW and NS components, are clearly seen in EW-NS component.

3. The Slichter mode triplets at 0.05 mHz

Disturbances by the Earth tide and the atmospheric pressure are a major problem in sub normal mode band. When we look at frequencies below 0.2 mHz in Figure 4, we can identify several horizontal lines which correspond to the period of hours. Lines at 12 and 24 hour periods in EW, NS and EW-NS are obviously the Earth tides. Atmospheric pressure fluctuations are often of semi-diurnal and diurnal, and horizontal line at 8 hour period in PRESS is their higher harmonics. Similarity between time-frequency variation of the strains (EW and NS components) and the atmospheric pressures (PRESS) suggest that the fluctuation of the atmospheric pressure is the major source of the noise in the strain records, for which the adiabatic compression and expansion of the observation vault is suggested as the driving mechanism (Morii, 2001).

In order to sanitize the strain records and the spectra, we correct both the Earth tides and the atmospheric pressure disturbances. We estimate tidal coefficients of major constituents with the Bayesian tidal analysis procedure (Tamura et al., 1991), for which we use the strain records re-sampled at 1 hour interval. With these estimates of tidal coefficients, theoretical Earth tide time series are computed with a 1 second interval (60 second interval for ESH), using TIDE4N (Tamura et al., 1991) and we subtract them from the extensometer strain records. The residual strains are filtered between 1 and 9 hour periods. We assume that the atmospheric pressures and the consequent disturbances on strains are in a linear relation with zero lag time, and estimate the atmospheric pressure disturbance by a linear regression between the residual strains and the atmospheric pressures which are likewise bandpass-filtered. Residue in this regression is the final product in the correction, the corrected strain. Figure 5 shows the running spectra of the corrected strains and the atmospheric pressures. By comparing Figures 4 and 5, significant reduction of the noise levels is obvious. Hereafter, the corrected strains are referred to as strains. Another advantage of these corrections is that we can isolate decaying spectral peaks; eigenoscillations whose eigenperiods are close to that of the higher harmonics of 4.8, 6 and 8 hours should survive the corrections. The amplitudes of the atmospheric pressure disturbances are of different magnitudes on the EW and NS components. Sensitivities of EW and NS components to the strain are not exactly the same in the frequencies of our present interest. We suspect that this is due to the local site factor, given that their mechanical values of the sensors are nearly equal.

Looking at Figure 5, we can identify triplet peaks in running spectra of NS and EW-NS components between 0.04 mHz to 0.06 mHz at around 72-th to 92-th hours. These peaks are equally spaced in frequencies and emergence of this triplet coincides with the occurrence of the 2004 Sumatra Andaman Island Earthquake. We therefore conclude (assume???) that this triplet

is the Slichter mode excited by this event.

Amplitudes of the triplet are of the same order as the noise level (Figure 5). From temporal decay, we have an estimate of apparent Q of the peaks to be approximately 15 as discussed in Supplement. Very low apparent Q should reflect high rate of energy loss by moving out of outer core fluid at ICB by the inner core translational motion. The initial and the peak spectral amplitudes of this event, which we obtain with the eigenfunction and eigenvalue using computer program of Saito (1967) and Takeuchi and Saito (1972) with the PREM by ignoring azimuth and distance dependent factors, are of the same order of magnitude as the observed spectral peak (Tables 3 and 4).

At a closer look at Figure 5, we recognize that frequencies of the observed triplet peaks are apparently consistent with theoretically predicted values for PREM (Figure 5), but when effect of Q is corrected (Kanamori and Anderson, 1977), they are actually more consistent with those for 1066A (Dahlen and Sailor, 1979); when Q is 15, the correction for Q changes eigenfrequencies approximately 21%. The major difference in the deep Earth in these two models is the amount of density jump at the ICB. While PREM has a jump of the 0.60 g/cm³, or approximately 5 % at the ICB, 1066A has a jump of the 0.87 g/cm³, approximately 7 %. Eigenfrequency of the gravity driven Slichter mode is sensitive to the density difference between the base of the outer core and the top of the inner core. Our finding that 1066A explains the observed frequencies of the Slichter mode suggests that the Earth has a 1066A like large density jump at the ICB. This is consistent with previous findings both from recent high and low frequency seismology.

Extensometer at Matsushiro Seismological Observatory (MSO) is one of the best strain stations in Japan. Yet the detected signal is close to our low detection limit. We also process the data at ESH, AMGS and BYB in a similar manner. However, we are not able to identify significant spectral peaks in this frequency band which stand above the noise level.

4. Summary and Concluding Remarks

By processing the strain records of the 2004 Sumatra-Andaman Islands event at Matsushiro Seismological Observatory, we identify triplet peaks of the Slichter mode in power spectra at approximately 0.05 mHz. Neither the Earth tides nor the atmospheric pressure disturbances explains this spectral signature, which emerged at the occurrence of the 2004 Sumatra-Andaman Islands earthquake. The observed frequencies favor the Earth model 1066A rather than PREM, which implies that the density jump at ICB is close to 1066A value.

Practical difficulties in dealing with long records, such as that we are forced to correct the Earth tides and the atmospheric pressure disturbances, are obviously an issue in our study. Similarly, improving frequency and amplitude resolution is not an easy task as this requires longer high quality time series data. Detecting a low Q and low frequency signal is consequently a hard task, which requires a higher quality data that we currently have.

Long period strain is currently not a popular quantity to be measured in solid Earth geophysics. Though observing small strain precisely is still a technical challenge itself, given that strains provide different information on movement of the Earth and thus processes in the Earth, we are sure that strains are a promising quantity to be looked at. We expect that new generation of the high quality strain records which includes Kamioka Razor strainmeter (Takemoto et al., 2004) and Plate Boundary Observatory in US together as well as superconducting gravimeter records of Global Geodynamics Project would lead us to observation of the Slichter mode with finer resolution.

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Earth density jump	Q	eigenfrequencies		
model	gr/cm ³	mHz (hour)		
1066A	0.87	0.055 (5.0)	0.061 (4.5)	0.067 (4.1)
1066A	0.87	15 0.046 (6.0)	0.051 (5.4)	0.057 (4.9)
PREM	0.60	0.047 (5.9)	0.051 (5.4)	0.057 (4.9)
PREM	0.60	15 0.039 (7.1)	0.043 (6.4)	0.048 (5.8)

Table 1. Triplet eigenfrequencies of the Slichter mode 1S1. Fourth and sixth rows are those corrected for Q. Whole Earth models of 1066A and PREM are from Gilbert and Dziewonski (1975) and Dziewonski and Anderson (1981), respectively. Degenerate singlet eigenfrequencies predicted for 1066A and PREM are from Dahlen and Sailor (1979) and Dahlen and Tromp (1998), respectively. Triplet eigenfrequencies are calculated using splitting parameters of Dahlen and Sailor (1979).

Code	Lat	Long	Burden	Comp.	Length	Rate	Strain/bit
AMGS	N34.88	E135.83	110m	M72.5W	40m	1 Hz	7.8×10^{-12}
MSO	N36.54	E138.21	100m	EW	100m	1 Hz	7.8×10^{-14}
ESH	N39.15	E141.34	60m	EW	35.7m	60 s	10^{-10}
BYB	N35.36	E137.30	1020m	N115E	1020m*	1 Hz	6.0×10^{-12}

Table 2. Four stations of continuous recording of crustal movements in Japan. AMGS is Amagase Station, Disaster Prevention Research Institute, Kyoto University, Kyoto Pref. MSO is Matsushiro Seismological Observatory, Japan Meteorological Agency, Nagano Pref. ESH is Esashi Earth tides station, Mizusawa VERA observatory, National Astronomical Observatory, Oshu, Iwate Pref. BYB is Byobusan geophysical station, Tono Research Institute of Earthquake Science, Mizunami, Gifu Pref. Code, Lat, Long, Burden, Comp, Length, Rate and Strain/bit in the first row denote station code, latitude, longitude, overburden thickness at the instrument site, highest S/N component, extensometer length (* bore hole depth), sampling rate and strain amplitude per bit, respectively.

Mode	Frequency	Period	Displ	Strain
	mHz	s	mm	strain
0S2	0.309	3233	0.28	4.3×10^{-11}
0S4	0.647	1545	0.35	5.5×10^{-11}
2S2	0.945*	1055	0.01	1.6×10^{-12}
1S1	0.0519*	1.95×10^4	0.9×10^{-5}	1.4×10^{-14}

Table 3. Initial amplitudes of 0S2, 0S4, 2S2 and 1S1 at MSO with a seismic moment of 1.2×10^{23} Nm (Stein and Okal, 2005) of the 2004 Sumatra-Andaman earthquake. Displ and Strain in the first row denote surface displacement and strain, respectively. Eigenfrequencies of 0S2 and 0S4 are from PREM (Dziewonski and Anderson, 1981). Those of 2S2 and 1S1 uncorrected for Q and initial amplitudes of 0S2, 0S4, 2S2 and 1S1 are computed by computer program of Saito (1967) with anisotropic Earth model PREM for the Harvard CMT solution of the 2004 Sumatra-Andaman Islands earthquake. Factors dependent of azimuth and distance are ignored.

Mode	Frequency	Q	Displ	Strain
	mHz		m·s	strain·s
0S2	0.309	463	130	2×10^{-5}
0S4	0.647	373	60	1×10^{-5}
2S2	0.945*	200-500	0.7-1.8	$(1-3) \times 10^{-7}$
1S1	0.0519*	30-50	$(1-2) \times 10^{-2}$	$(3-4) \times 10^{-9}$

Table 4. Peak spectral amplitudes of 0S2, 0S4, 2S2 and 1S1 predicted for the 2004 Sumatra-Andaman earthquake. Q of 0S2 and 0S4 are from PREM. Q of 2S2 and 1S1 are tentatively assumed. Pertinent data are the same as Table 3.

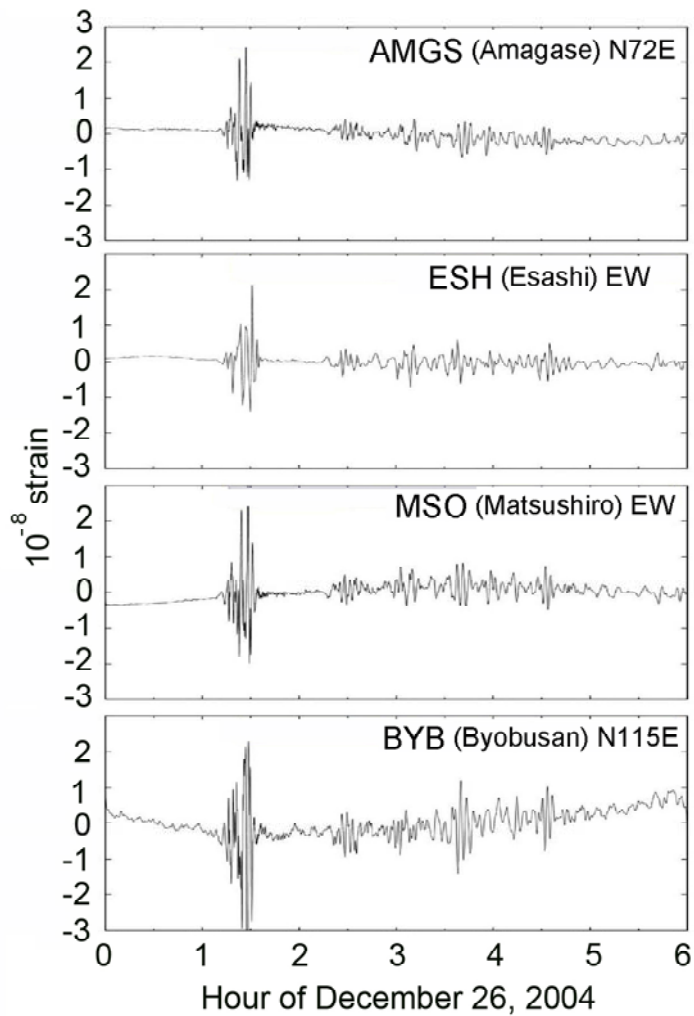


Fig.1 Six hour strain records of December 26, 2004, at AMGS, MSO, ESH and BYB, due to the Sumatra-Andaman earthquake (Mw9.0). Time interval are 1 second except 60 seconds at ESH. They are lowpass-filtered at 100 seconds except unfiltered at ESH. Horizontal axis is hour of December 26, 2004. Vertical axis is 10^{-8} strain. Noise levels at these stations are lowest among stations of continuous recording of crustal movements in Japan. The records are the highest S/N component records at the respective stations.

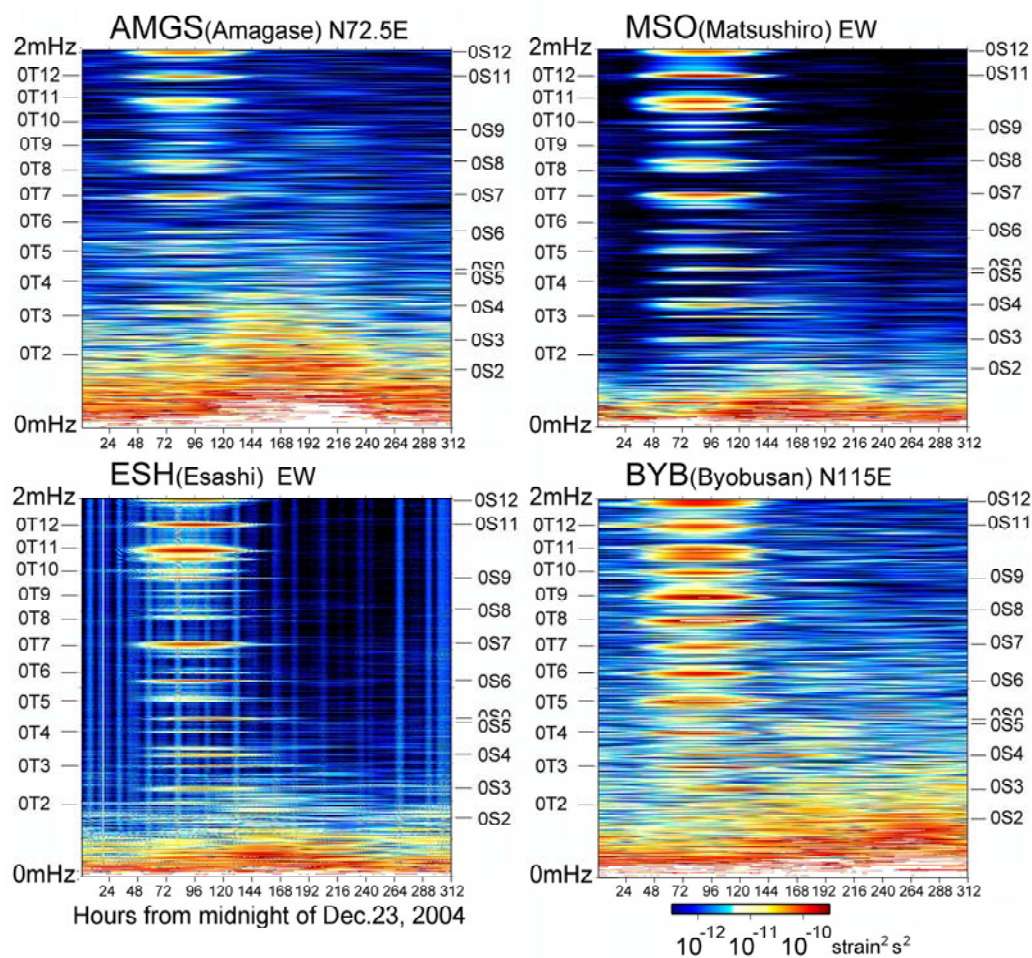


Fig.2 Running power spectra for frequencies below 2 mHz (500 s) of strain records at four stations for the respective components shown in Figure 1. Vertical axis is frequency from 0 mHz to 2 mHz. Horizontal axis is hour from December 23, 2004, to January 5, 2005. A power spectrum in the first column on the left side of each panel is that for 6.07 days from 00:00:00 of December 20 to 01:38:08 of December 26. Locations of eigenfrequencies of spheroidal (0Sn) and toroidal (0Tn) modes are indicated on the right and left sides of each panel, respectively.

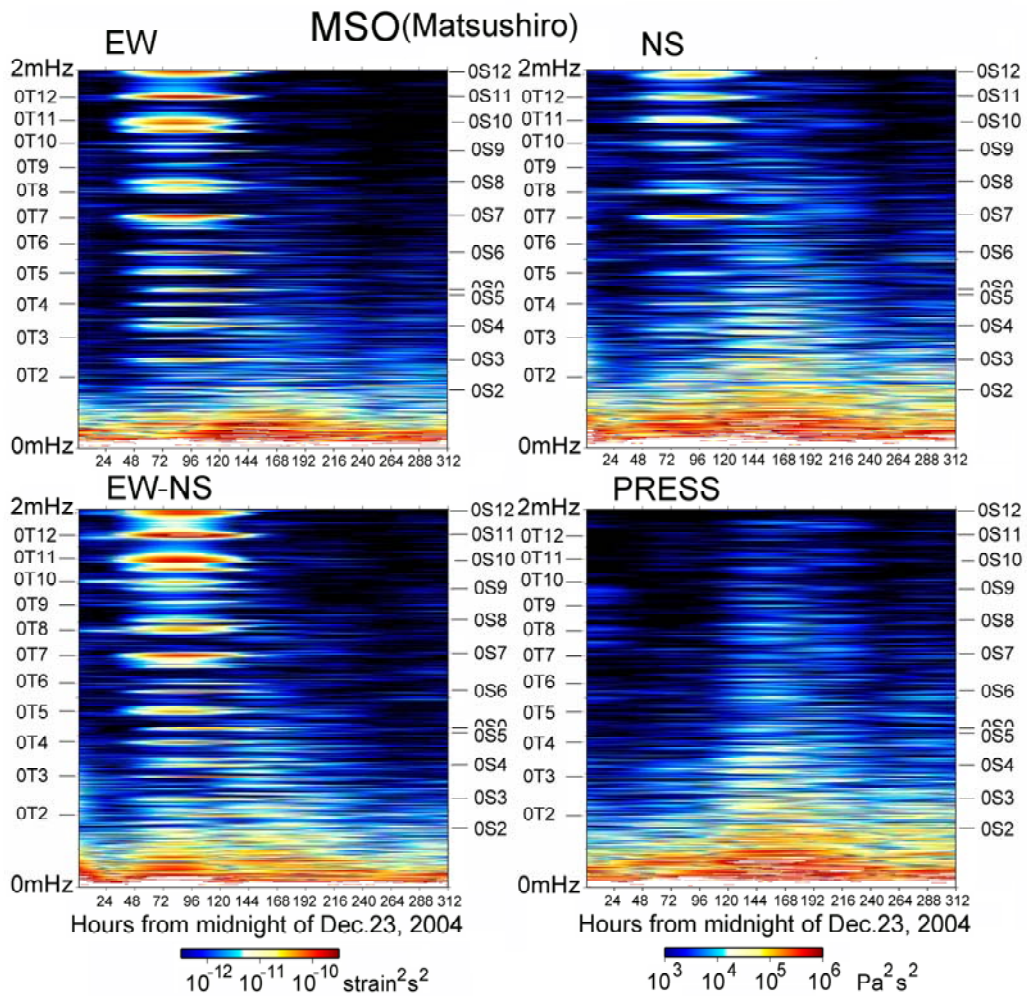


Fig.3 Running power spectra for frequencies below 2 mHz of strain records of 2 component (EW and NS) extensometer and atmospheric pressures (PRESS) in the observation vault at MSO due to the Sumatra-Andaman Islands earthquake. EW-NS denotes running power spectra of difference between EW and NS component strains. Pertinent data are the same as in Fig.2.

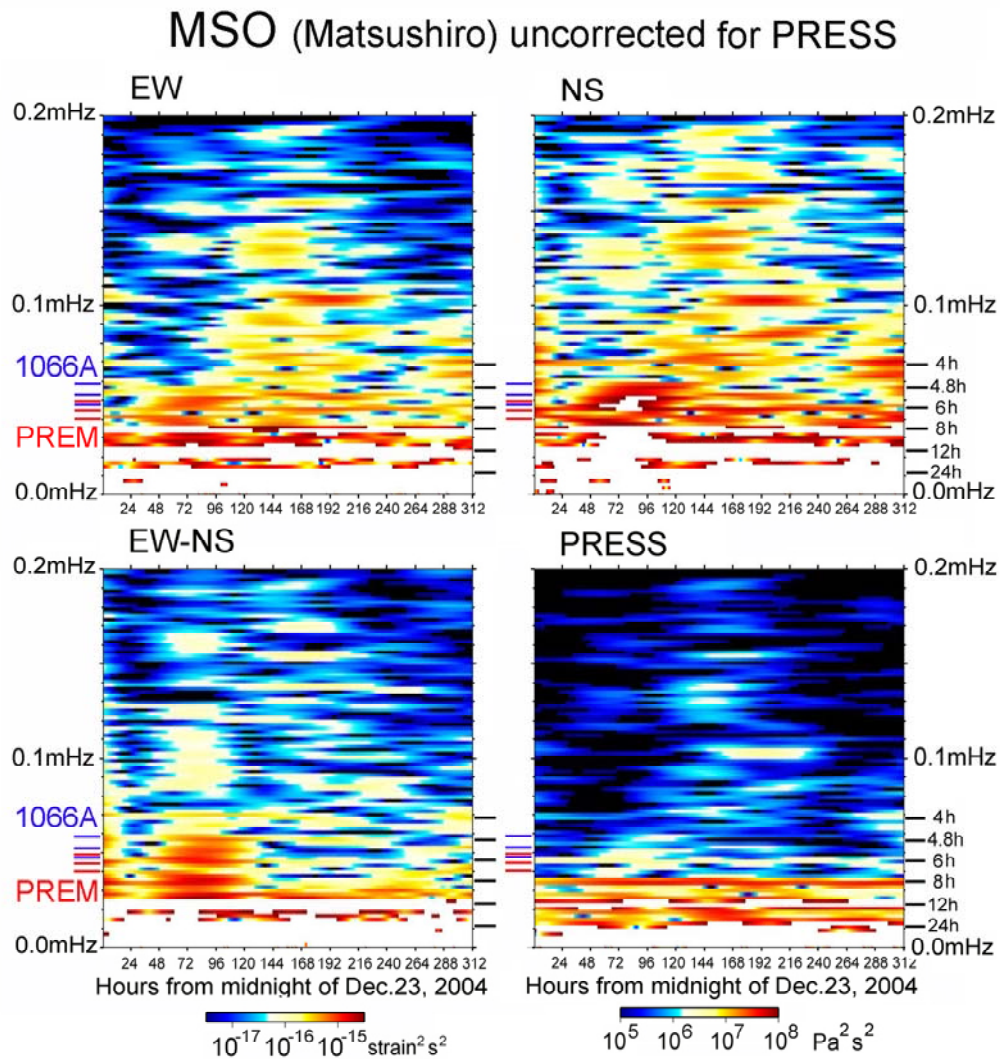


Fig.4 Blow-up for frequencies below 0.2 mHz (5000 s) of the running power spectra of extensometer records at MSO due to the 2004 Sumatra-Andaman Islands earthquake. Tick marks on the right sides denote periods in hour. White and black colors in the running spectra denote amplitudes larger than $3 \times 10^{-15} \text{ strain}^2 \text{ s}^2$ of upper bound and smaller than $3 \times 10^{-18} \text{ strain}^2 \text{ s}^2$ of lower bound of common color scale, respectively. The triplet eigenfrequencies theoretically predicted for PREM (Dahlen and Tromp, 1998) and 1066A (Dahlen and Sailor, 1978) corrected for Q are indicated by red and blue tick marks on the left side of each panel, respectively. Pertinent data are the same as in Figures 2 and 3.

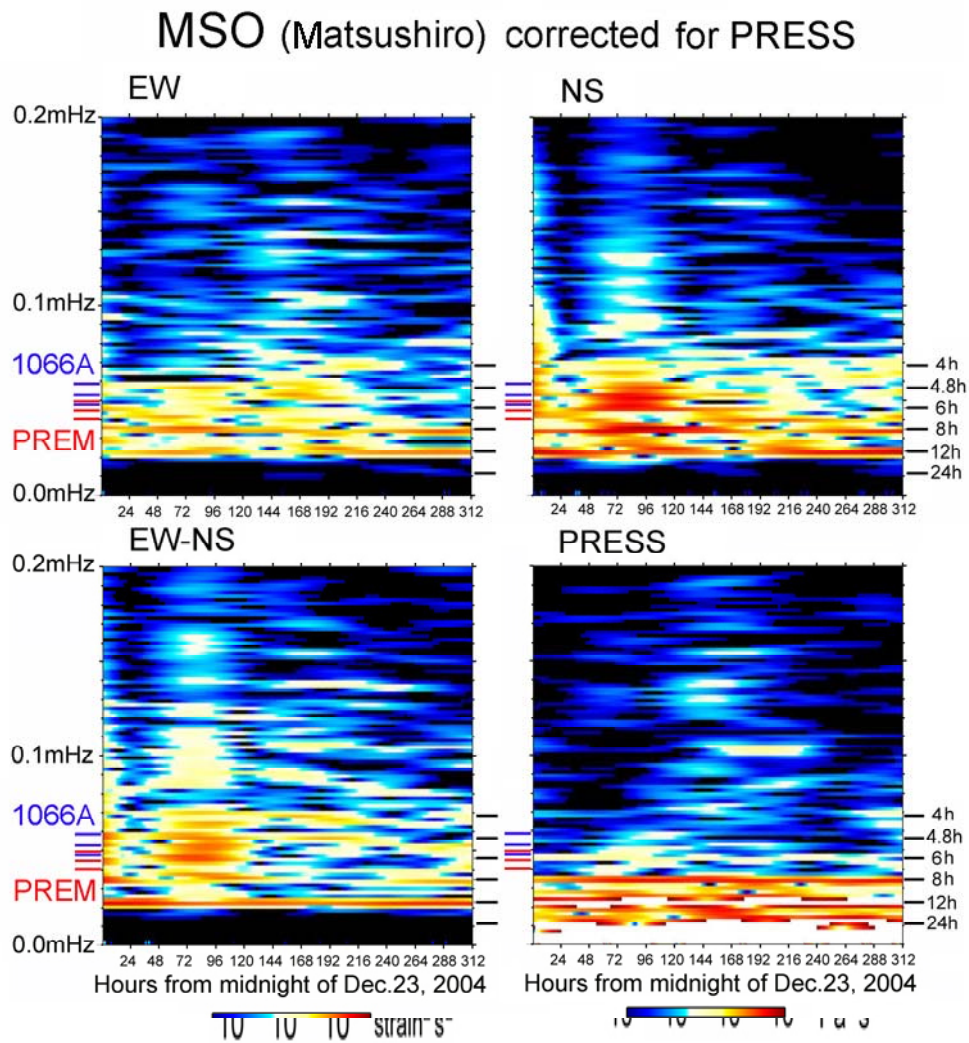
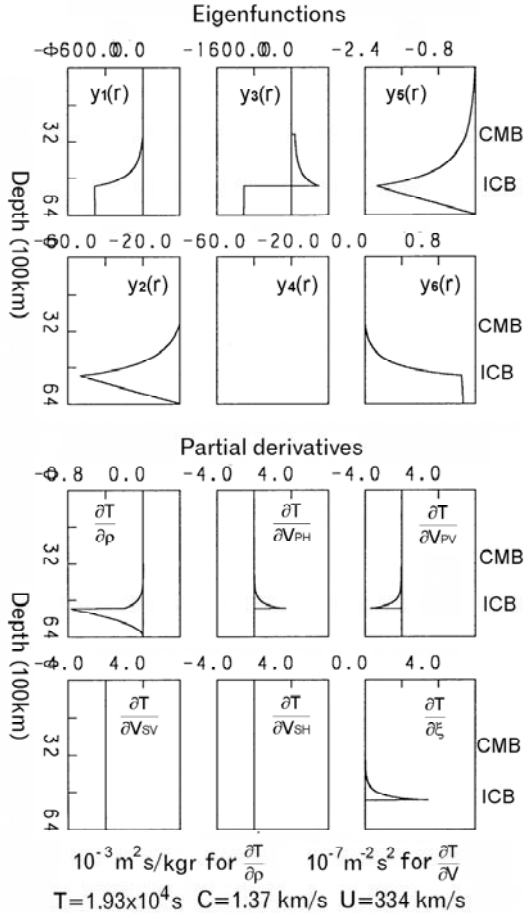


Fig.5 Blow-up for frequencies below 0.2 mHz of the running power spectra of extensometer strain records at MSO due to the 2004 Sumatra-Andaman Islands earthquake corrected for Earth tides and atmospheric pressures. Pertinent data are the same as in Figures 2-4.



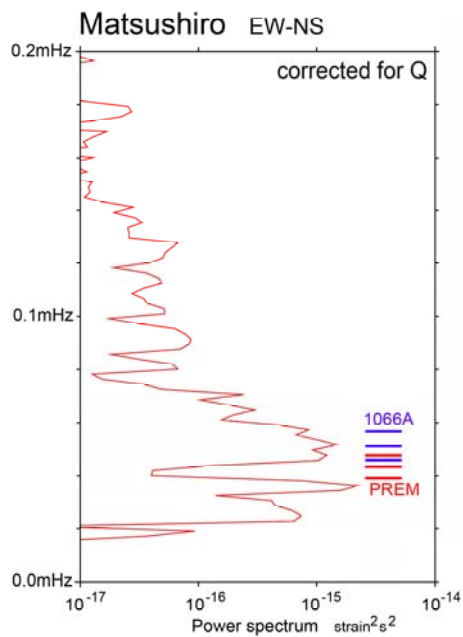
Supplementary Figure S1. Eigenfunctions and partial derivatives of the Slichter mode. T , C and U are an eigenperiod, a phase and a group velocities, respectively. Eigenfunctions of $y^2(r)$ to $y^6(r)$ are normalized by $y^1(r=r_0)$ where r and r_0 are a distance from the center and a radius of the Earth, respectively. $y^1(r)$ to $y^6(r)$ are related to surface displacement u_r , u_θ and u_ϕ and stresses σ_{rr} , $\sigma_{r\theta}$ and $\sigma_{\phi r}$ as

$$u_r = y^1(r)Y_n(\theta, \phi), \quad u_\theta = y^3(r) \frac{\partial Y_n(\theta, \phi)}{\partial \theta}, \quad u_\phi = y^3(r) \frac{1}{\sin \theta} \frac{\partial Y_n(\theta, \phi)}{\partial \phi}, \quad \psi = y^5(r)Y_n(\theta, \phi), \quad \sigma_{rr} = y^2(r)Y_n(\theta, \phi),$$

$$\sigma_{\phi r} = y^4(r) \frac{1}{\sin \theta} \frac{\partial Y_n(\theta, \phi)}{\partial \phi}, \quad \sigma_{r\theta} = y^4(r) \frac{\partial Y_n(\theta, \phi)}{\partial \theta},$$

where (r, θ, ϕ) are spherical polar coordinates.

$Y_n(\theta, \phi)$ is spherical harmonics of angular order of n . ρ and ξ are density and anisotropy parameter, respectively. V_P and V_S denote P and S wave velocities, respectively. Subscripts H and V denote horizontal and vertical propagation directions, respectively. CMB and ICB denote the core-mantle and the inner-outer core boundaries, respectively. $\partial T / \partial \rho$ displays how sensitive the eigenperiod is to inner core density.



Supplementary Figure S2. Power spectrum of EW-NS component strain at MSO of the 2004 Sumatra-Andaman Islands earthquake for the time domain window of 6.07 days from 00:00:00.0 of Dec. 23 to 01:38:08.0 of Dec. 29, 2004. This corresponds to that at 72-th hour in EW-NS of Fig.5. Blue and red short bars indicate the Slichter mode triplet eigenfrequencies theoretically predicted for 1066A (Dahlen and Sailor, 1978) and for PREM (Dahlen and Tromp, 1998) corrected for Q of 15, respectively.